Formulae

For CIE Mathematics 4024

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NUMBER

Natural Numbers: Numbers which are used for counting purpose are called natural numbers.
Ex: 1, 2, 3, 4, ...100, ...

Whole Numbers: Natural numbers including 0 are called Whole Numbers.
Ex: 0, 1, 2, 3, 4, ...

Integers: Positive natural numbers, negative natural numbers along with 0 are called integers.
Ex.: ...., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

Rational Numbers: Numbers which are in the form of $\frac{p}{q}$ $(q \neq 0)$ where p and q are positive or negative whole numbers are called rational numbers.
Ex: $\frac{1}{2}, \frac{3}{4}, \frac{-5}{7}, \frac{49}{-56}$ ....

Irrational Numbers: Numbers like $\sqrt{2}$, π cannot be expressed as rational numbers. Such types of numbers are called as irrational numbers.
Ex: $\sqrt{5}$, $\sqrt{17}$, ...

Terminating Decimals
These are decimal numbers which stop after a certain number of decimal places.
For example, 7/8 = 0.875, is a terminating decimal because it stops (terminates) after 3 decimal places.

Recurring Decimals
These are decimal numbers which keep repeating a digit or group of digits; for example 137/259,=0.528 957 528 957 528 957 ...., is a recurring decimal. The six digits 528957 repeat in this order. Recurring decimals are written with dots over the first and last digit of the repeating digits, e.g. $0.\dot{5}28\ 95\dot{7}$

- The order of operations follows the BODMAS rule:

  Brackets open
  Divide
  Multiply
  Add
  Subtract

- **Even numbers:** numbers which are divisible by 2, eg, 2, 4, 6, 8, ...
- **Odd numbers:** numbers which are not divisible by 2, eg; 1, 3, 5, 7 ...

- **Real numbers** are made up of all possible *rational* and *irrational* numbers.
- An **Integer** is a whole number.
- A **prime number** is divisible only by itself and by one (1). 1 is not a prime number. It has only two factors. 1 and the number itself.
- The exact value of rational number can be written down as the ratio of two whole numbers.
- The exact value of an **Irrational** number cannot be written down.
- A **square number** is the result of multiplying a number by itself.
  Ex: $1^2, 2^2, 3^2$, ... i.e. 1, 4, 9, ...
- A **cube number** is the result of multiplying a number by itself three times.
  Ex: $1^3, 2^3, 3^3$, ... i.e. 1, 8, 27,.........
- The **factors** of a number are the numbers which divide exactly into two.
  eg. Factors of 36
  1, 2, 3, 4, 6, 9, 12, 18
- **Multiples** of a number are the numbers in its times table.
  eg. Multiples of 6 are 6, 12, 18, 24, 30, ...

Significant figures;
Example;
8064 = 8000 (correct to 1 significant figures)
8064 = 8100 (correct to 2 significant figures)
8064 = 8060 (correct to 3 significant figures)
0.00508 =0.005 (correct to 1 significant figures)
0.00508 = 0.0051 (correct to 2 significant figures)
2.00508 = 2.01 (correct to 3 significant figures)

Decimal Places
Example
0.0647 = 0.1 (correct to 1 decimal places)
0.0647 = 0.06 (correct to 2 decimal places)
0.0647 = 0.065 (correct to 3 decimal places)
2.0647 = 2.065 (correct to 3 decimal places)

Standard Form:
The number $A \times 10^n$ is in standard form when $1 \leq A < 10$ and n is a positive or negative integer.
Eg: $2400 = 2.4 \times 10^3$
$0.0035 = 3.5 \times 10^{-3}$

Conversion Factors:

Length:
1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

km means kilometer
m means meter
cm means centimeter
mm means millimeter

Mass:
1 kg = 1000 gm where kg means kilogram
1 gm = 1000 mgm gm means gram
1 tonne = 1000 kg mgm means milligram

Volume:
1 litre = 1000 cm^3
1 m^3 = 1000 litres
1 kilo litre = 1000 litre
1 dozen = 12

Time:
1 hour = 60 minutes = 3600 seconds
1 minute = 60 seconds.
1 day = 24 hours
1 year = 12 months
= 52 weeks
= 365.25 days.

1 week = 7 days
1 leap year = 366 days
1 light year = 9.46×10^{12} km.

Percentages:
- Percent means per hundred.
- To express one quantity as a percentage of another, first write the first quantity as a fraction of the second and then multiply by 100.
- Profit = S.P. – C.P.
- Loss = C.P. – S.P.
- Profit percentage = $\frac{SP-CP}{CP} \times 100$
- Loss percentage = $\frac{CP-SP}{CP} \times 100$

where CP = Cost price and SP = Selling price

Simple Interest:

To find the interest:
- $I = \frac{PRT}{100}$ where
  P = money invested or borrowed
  R = rate of interest per annum
  T = Period of time (in years)

To find the amount:
- $A = P + I$ where A = amount

Compound Interest:

$$A = p\left(1 + \frac{r}{100}\right)^n$$

Where,
A stands for the amount of money accruing after n year.
P stands for the principal Amount
R stands for the rate per cent per annum
n stands for the number of years for which the money is invested.

Speed, Distance and Time:

- Distance = speed x time
- Speed = $\frac{distance}{time}$
- Time = $\frac{distance}{Speed}$
- Average speed = $\frac{total\ distance}{total\ time}$

- Units of speed: km/hr, m/sec
- Units of distance: km, m
- Units of time: hr, sec
- km / hr × $\frac{5}{18}$ = m / sec
- m / sec × $\frac{18}{5}$ = km / hr

ALGEBRA

Quadratic Equations:

An equation in which the highest power of the variable is 2 is called quadratic equation. Thus $ax^2 + bx + c = 0$ where a, b, c are constants and $a \neq 0$ is a general equation.

Solving quadratic equations:

We can solve quadratic equation by method of,

a) Factorization
b) Using the quadratic formula
c) Completing the square

(a) Solution by factors:

Consider the equation c × d = 0, where c and d are numbers. The product c × d can only be zero if either c or d (or both) is equal to zero.

i.e. c = 0 or d = 0 or c = d = 0.

(b)Solution by formula:

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(c) Completing the square

- Make the coefficient of x^2, i.e. a = 1
- Bring the constant term, i.e. c to the right side of equation.
- Divide coefficient of x, i.e. by 2 and add the square i.e. $(\frac{b}{2})^2$ to both sides of the equation.
- Factorize and simplify answer

Expansion of algebraic expressions

- $a(b+c) = ab + ac$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 + b^2 = (a + b)^2 - 2ab$
- $a^2 - b^2 = (a + b)(a - b)$

Factorization of algebraic expressions

- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^2 - b^2 = (a+b)(a-b)$

Ordering:

= is equal to
≠ is not equal to
> is greater than
≥ is greater than or equal to
< is less than
≤ is less than or equal to

Variation:

Direct Variation:	Inverse Variation:
y is proportional to x	y is inversely proportional to x
$y \propto x$	$y \propto \frac{1}{x}$
$y = kx$	$y = \frac{k}{x}$

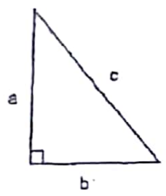
MENSURATION

PYTHAGORAS' THEOREM

For all the right angled triangles " the square ofof ıe hypotenuse is equal to the sum of the squares on the other two sides"

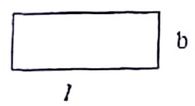
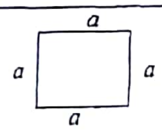
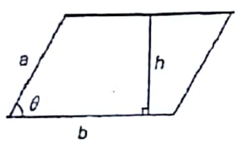
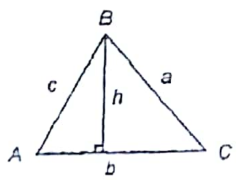
$c^2 = a^2 + b^2$

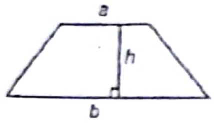
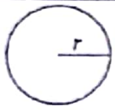
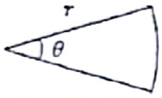
Note:
If three sides are given, and if the square of the larger side is equal to the sum of the squaresof the other two sides, then these are the sides of a right angled triangle.



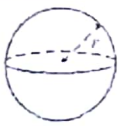
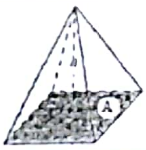
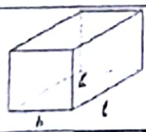
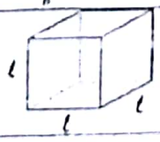
$c = \sqrt{a^2 + b^2}$ $b = \sqrt{c^2 - a^2}$ $a = \sqrt{c^2 - b^2}$

Area and Perimeter:

Figure	Diagram	Area	Perimeter
Rectangle		Area = $l \times b$	perimeter = $2(l + b)$
Square		Area = side × side = $a \times a$	perimeter = 4 × side = $4 \times a$
Parallelogram		Area = $b \times h$ Area = $ab \sin\theta$ where a, b are sides and θ is the included angle	perimeter = $2(a + b)$
Triangle		Area = $\frac{1}{2} \times base \times height$ Area = $\frac{1}{2} ab \sin C$ $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$	perimeter = $a + b + c$

Trapezium		Area $=\frac{1}{2}(a+b)h$	perimeter = Sum of all sides
Circle		Area = πr^2	circumference = $2\pi r$
Semicircle		Area = $\frac{1}{2}\pi r^2$	perimeter = $\frac{1}{2}\pi d + d$
Sector		Area = $\pi r^2 \times \frac{\theta}{360°}$	length of an arc = $2\pi r \times \frac{\theta}{360}$

Surface Area and Volume:

Figure	Diagram	Surface Area	Volume
Cylinder		curved surface area = $2\pi rh$ total surface area = $2\pi r(h+r)$	Volume = $\pi r^2 h$
Cone		curved surface area = $\pi r l$ where $l = \sqrt{(r^2+h^2)}$ total surface area = $\pi r(l + r)$	Volume = $\frac{1}{3}\pi r^2 h$
Sphere		Surface area = $4\pi r^2$	Volume = $\frac{4}{3}\pi r^3$
Pyramid		Base area + area of the shapes in the sides	Volume = $\frac{1}{3}$ × base area × perpendicular height
Cuboid		Surface area = $2(lb + bh + lh)$	Volume = $l \times b \times h$
Cube		Surface area = $6l^2$	Volume = l^3
Hemisphere		Curved surface area = = $2\pi r^2$	Volume = $\frac{2}{3}\pi r^3$

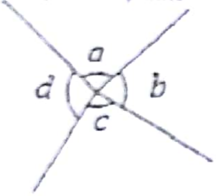
GEOMETRY

(a) Angles on a straight line

The angles on a straight line add up to 180°.

$x + y + z = 180°$

(b) Angle at a point



The angles at a point add up to 360°.

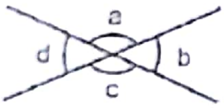
$a + b + c + d = 360°$

(c) **Vertically opposite angles**

If two straight line intersect, then

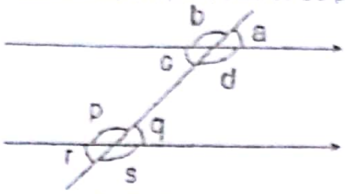
$a = c$

$b = d$ (Vert,opp.∠s)



Parallel Lines:

When lines never meet, no matter how far they are extended, they are said to be parallel.

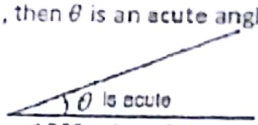


- Vertically opposite angles are equal.
  a = c; b = d; p = s and q = r
- **Corresponding angles are equal.**
  $a = q;\ b = p;\ c = r$ and $d = s$
- Alternate angles are equal.
  c = q and d = p.
- Sum of the angles of a triangle is 180°.
- Sum of the angles of a quadrilateral is 360°.
- Interior or allied angles, their sum is equal to 180°, p + c = 180, q + d = 180.

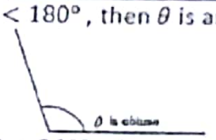
Types of angles

Given an angle, if

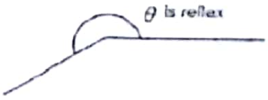
$\theta < 90°$, then θ is an acute angle



$90° < \theta < 180°$, then θ is an obtuse angle



$180° < \theta < 360°$, then θ is an reflex angle

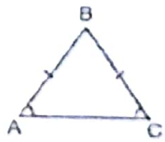


Triangles

Different types of triangles:

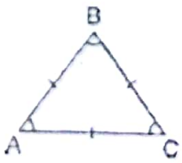
1. An isosceles triangle has 2 sides and 2 angles the same.

   AB = AC
   BÂC = BĈA



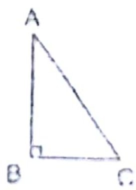
2. An equilateral triangle has 3 sides and 3 angles the same.

   AB = BC = CA and ABC = BCA = CAB



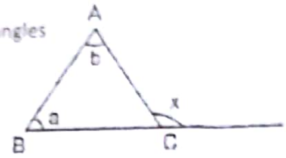
3. A triangle in which one angle is a right angle is called the right angled triangle.

   = 90°



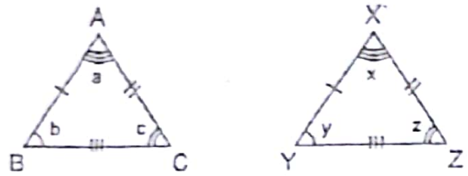
- The sum of the angles of a triangle is equal to 180°.
- In every triangle, the greatest angle is opposite to the longest side. The smallest angle is opposite to the shortest side.
- Exterior angle is equal to the Sum of the non-adjacent interior opposite angles

$x = a + b$



Congruent Triangles:

Two triangles are said to be congruent if they are equal in every aspect.

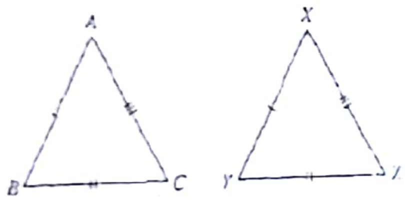


AB = XY	∠a = ∠x
BC = YZ	∠b = ∠y
AC = XZ	∠c = ∠z

Congruency tests

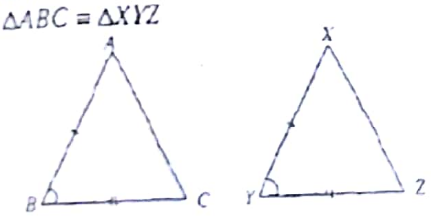
(a) SSS property

If $AB = XY$, $BC = YZ$ and $AC = XZ$ then $\Delta ABC \equiv \Delta XYZ$



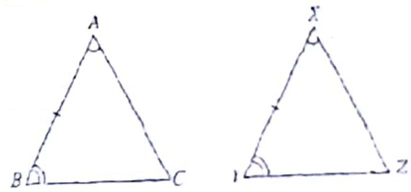
(b) SAS property

If $AB = XY$, $BC = YZ$ and $\angle B = \angle Y$ then $\Delta ABC \equiv \Delta XYZ$



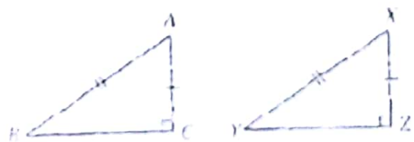
(c) ASA property

If $AB = XY$, $\angle A = \angle X$ and $\angle B = \angle Y$, then $\Delta ABC \equiv \Delta XYZ$



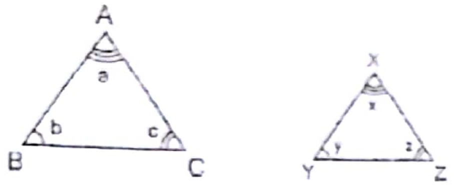
(d) RHS property

If $\angle B = \angle Y = 90°$ (Right angle), $AC = XZ$ (Hypotenuse) and $AB = XY$ (one other side), then $\Delta ABC \equiv \Delta XYZ$.



Similer Triangles.

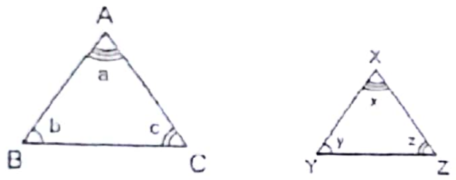
If two triangles are similar then they have a pair of corresponding equal angles and the three ratios of corresponding sides are equal.



$\angle a = \angle x$; $\angle b = \angle y$ and $\angle c = \angle z$ $\qquad \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

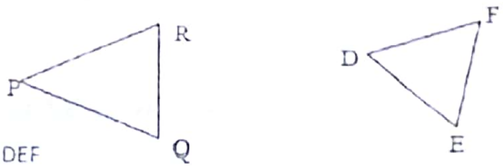
If you can show that one of the following conditions is true for two triangles, then the two triangles are similar.

i) The angles of one triangle are equal to the corresponding angles of the other triangle.



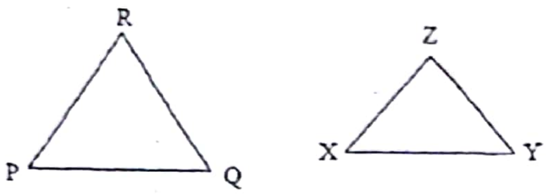
Δ ABC is similar to Δ XYZ because $\angle a = \angle x$; $\angle b = \angle y$ and $\angle c = \angle z$

ii) The ratio of corresponding sides is equal.



If $\frac{PQ}{DE} = \frac{PR}{DF} = \frac{QR}{EF}$ then Δ PQR is similar to Δ DEF

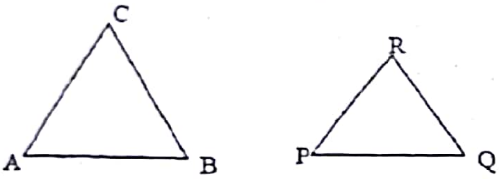
iii) The ratios of the corresponding sides are equal and the angles between them are equal.



Δ PQR is similar to Δ XYZ (if, for eg: $\angle P = \angle X$ and $\frac{PQ}{XY} = \frac{PR}{XZ}$)

Areas of Similar Triangles:

The ratio of the areas of similar triangles is equal to the ratio of the square of corresponding sides.



$$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Polygons:

i) The exterior angles of a polygon add up to 360°.

ii) The sum of the interior angles of a polygon is $(n-2) \times 180°$ where n is the number of sides of the polygon.

iii) A regular polygon has equal sides and equal angles.

iv) If the polygon is regular and has n sides, then each exterior angle = $\frac{360}{n}$

v) Each interior angle of a regular n-sided poly- gon = $\frac{(n-2)\times 180°}{n}$

vi)

3 sides = triangle	4 sides = quadrilateral	5 sides = pentagon
6 sides = hexagon	7 sides = heptagon	8 sides = octagon
9 sides = nonagon	10 sides = decagon	

Similar Solids

If two objects are similar and the ratio of corresponding sides is k, then

- the ratio of their areas is k^2.
- the ratio of their volumes is k^3.

Length

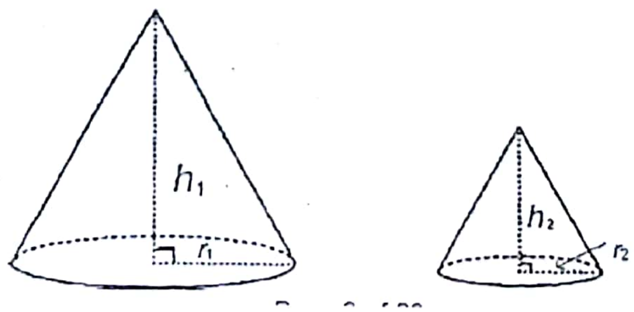
$$\frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2}$$

Area

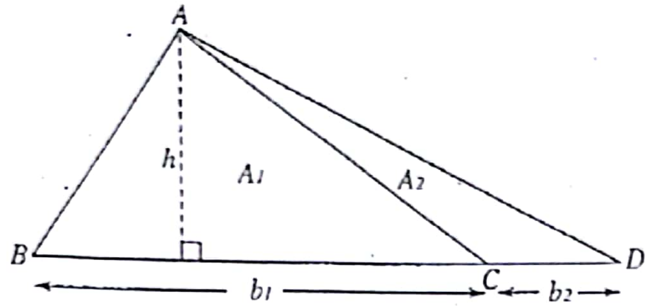
$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{h_1^2}{h_2^2}$$

Volume

$$\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \frac{h_1^3}{h_2^3}$$



- **Triangles having the same height**



$$\text{Area of } \Delta ABC = \tfrac{1}{2} b_1 h$$

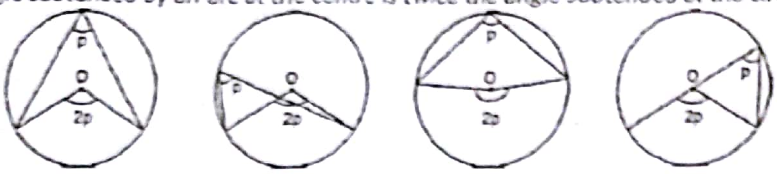
$$\text{Area of } \Delta ACD = \tfrac{1}{2} b_2 h$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ACD} = \frac{\tfrac{1}{2} b_1 h}{\tfrac{1}{2} b_2 h}$$

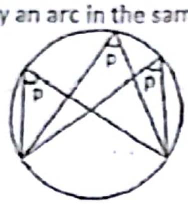
$$\therefore \frac{A_1}{A_2} = \frac{b_1}{b_2}$$

CIRCLE

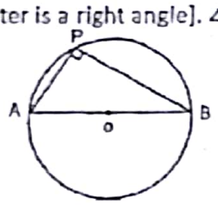
- The angle subtended by an arc at the centre is twice the angle subtended at the circumference



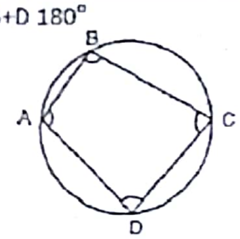
- Angles subtended by an arc in the same segment of a circle are equal.



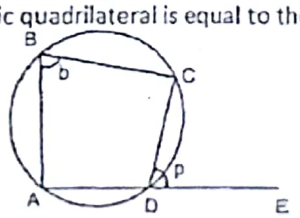
- The angle in a semi-circle is a right angle. [or if a triangle is inscribed in a semi-circle the angle opposite the diameter is a right angle]. $\angle APB = 90°$



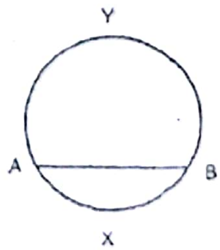
- **Note.** The hypotenuse of a right angle triangle, is always the diameter of the circle
- Opposite angles of a cyclic quadrilateral add up to 180° (supplementary). The corners touch the circle. A+C = 180°, B+D 180°



- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.$(b = p)$



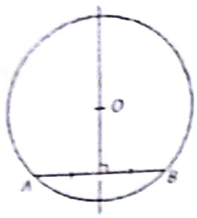
Chord of a circle:
A line joining two points on a circle is called a chord.
The area of a circle cut off by a chord is called a segment.
AXB is the minor arc and AYB is the major arc.



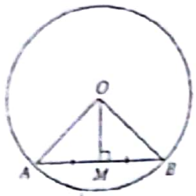
a) The line from the centre of a circle to the mid-point of a chord bisects the chord at right angles.
b) The line from the centre of a circle to the mid-point of a chord bisects the angle subtended by the chord at the centre of the circle.

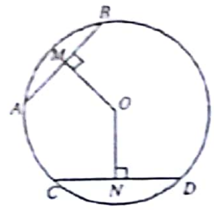
CIE O Level Mathematics June 2017 Exams
Munir Ahmad Khan (03004230019)

- Given a chord *AB*, i.e. a straight line joining two points *A* and *B* on the circumference of a circle, the **perpendicular bisector** of the chord *AB* always passes through the centre of the circle, *O*.
  Conversely the perpendicular from the centre, *O* to the chord *AB* bisects the chord.



- If *M* is the mid-point of the chord *AB*, then *OM* is perpendicular to *AB*, i.e.
  $\angle OMA = \angle OMB = 90°$
  Conversely if *OM* is perpendicular to *AB*, then *AM* = *MB*, i.e. *M* is the mid-point of *AB*.



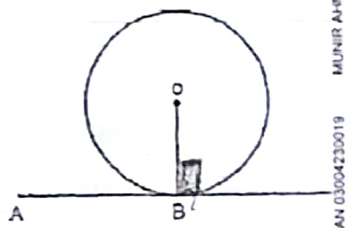


- Equal chords of a circle, *AB* and *CD* are equidistant from the centre *O*, i.e.
  $OM = ON$

Tangents to a Circle:

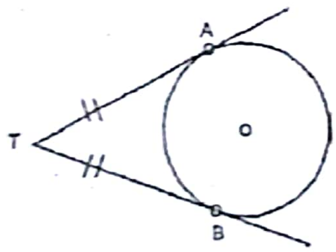
- The angle between a tangent and the radius drawn to the point of contact is 90°.

  $\angle ABO = 90^{0}$



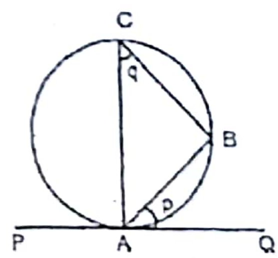
- From any point outside a circle just two tangents to the circle may be drawn and they are of equal length.

  TA = TB



- **Alternate Segment Theorem**
  The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

  $\angle QAB = \angle ACB$ $(p = q)$



FORMULAE AND IMPORTANT NOTES

1. The three laws of indices:

$x^a \cdot x^b = x^{a+b}$, $x^a \div x^b = \frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}$

$(x^a)^b = x^{ab}$

2. $x^a \cdot y^a = (xy)^a$, $x^a \div y^a = \frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$

3. Let a and b be positive integers

(i) $x^a = x \cdot x \cdot x \ldots$ to a factor

(ii) $x^0 = 1$

(iii) $x^{-1} = \frac{1}{x}$, $\left(\frac{1}{x}\right)^{-1} = x$, $\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$

$x^{-a} = \frac{1}{x^a}$, $\left(\frac{1}{x}\right)^{-a} = x^a$, $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a = \frac{y^a}{x^a}$

$\frac{x^a}{y^b} = \frac{y^{-b}}{x^{-a}} = x^a y^{-b} = \frac{1}{x^{-a} y^b}$

(iv) $x^{\frac{1}{a}} = \sqrt[a]{x}$, $x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$

$x^{-\frac{a}{b}} = \frac{1}{x^{\frac{a}{b}}}$

4. If $y^b = x^a$ then $y = x^{\frac{a}{b}}$

5. $10^{-a} = \frac{1}{10^a}$ = the digit 1 that is a places after the decimal point

$10^0 = 1$

10^a = the digit 1 followed by a zero(s)

A surd is an irrational root of a number.

6. (i) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(ii) $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = (\sqrt{a})^2 = a$

(iii) $\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{a \div b} = \sqrt{\frac{a}{b}}$

(iv) $\sqrt{a^2 b} = a\sqrt{b}$

(v) $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

(vi) $\frac{a\sqrt{b}}{c} = \frac{a}{c}\sqrt{b}$

(vii) $\frac{a}{\sqrt{a}} = \sqrt{a}$

(viii) $\frac{\sqrt{a}}{a} = \frac{1}{\sqrt{a}}$

(ix) $\frac{1}{a}\sqrt{b} = \frac{\sqrt{b}}{a} = \sqrt{\frac{b}{a^2}}$

(x) $\frac{a\sqrt{b}}{\sqrt{c}} = a\sqrt{\frac{b}{c}}$

(xi) $\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}}$

(xii) $\sqrt{a^{2b}} = a^b$

7. (i) $\sqrt{a^{2b} \times c} = a^b \times \sqrt{c}$

(ii) $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ unless one or both of a and b are zero

(iii) $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$ unless b is zero

(iv) $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$ $a\sqrt{b} + a\sqrt{c} = a\left(\sqrt{b} + \sqrt{c}\right)$

(v) $a\sqrt{b} - c\sqrt{b} = (a-c)\sqrt{b}$ $a\sqrt{b} - a\sqrt{c} = a\left(\sqrt{b} - \sqrt{c}\right)$

(vi) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$

(vii) $\left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b$

(viii) $(\sqrt{a} + b)(\sqrt{a} - b) = a - b^2$

(ix) $\left(\sqrt{a} + \sqrt{b}\right)^2 = \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = a + b + 2\sqrt{ab}$

(x) $\left(a + \sqrt{b}\right)^2 = \left(a + \sqrt{b}\right)\left(a + \sqrt{b}\right) = a^2 + b + 2a\sqrt{b}$

(xi) $(\sqrt{a} + b)^2 = (\sqrt{a} + b)(\sqrt{a} + b) = a + b^2 + 2b\sqrt{a}$

(xii) $\left(\sqrt{a} - \sqrt{b}\right)^2 = \left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a + b - 2\sqrt{ab}$

INDICES:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{-n} = \dfrac{1}{a^n}$
- $(a \times b)^m = a^m \times b^m$
- $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
- $(\sqrt[n]{a})^m = a^{m/n}$
- $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- $(\sqrt{a})^2 = a$

Solving Inequalities:

When we multiply or divide by a negative number the inequality is reversed.

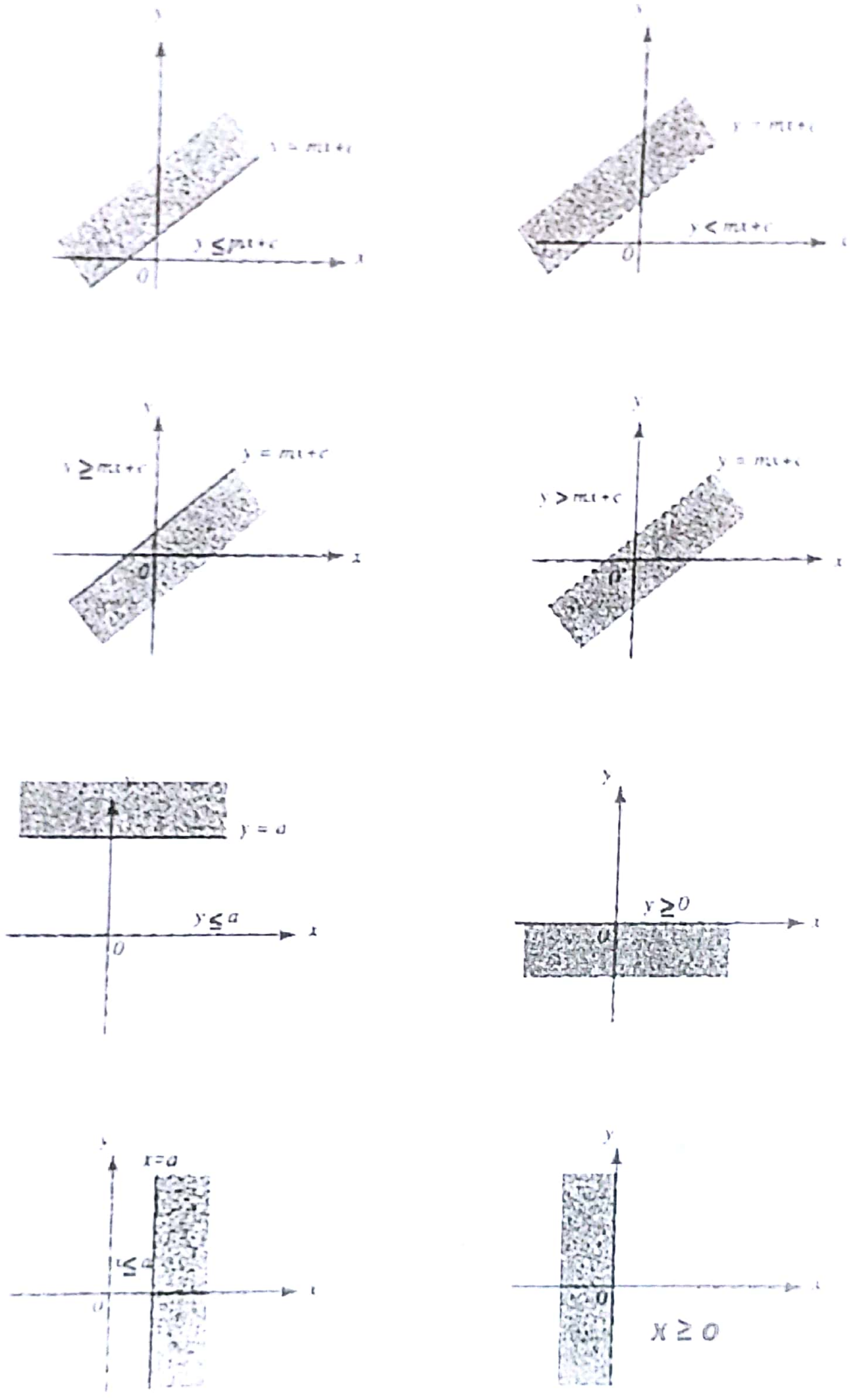
Eg: $4 > -2$

By multiplying by -2 $[4(-2) < (-2)(-2)]$

$-8 < +4$

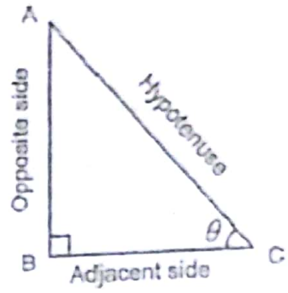
• Graphical Representation of linear Inequalities

The diagrams below show some inequalities that define the un-shaded region



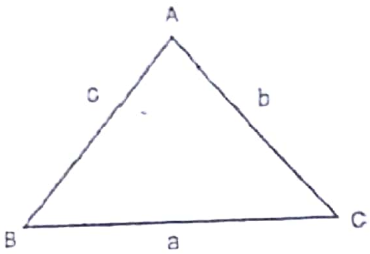
TRIGONOMETRY

Let ABC be a right angled triangle, where $\angle B = 90^\circ$



- $\sin\theta = \dfrac{\text{Opposite Side}}{\text{Hypotenuse}} = \dfrac{O}{H}$
- $\cos\theta = \dfrac{\text{Adjacent Side}}{\text{Hypotenuse}} = \dfrac{A}{H}$
- $\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent Side}} = \dfrac{O}{A}$

SOH CAH TOA



Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:

To find the length of a side:

- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

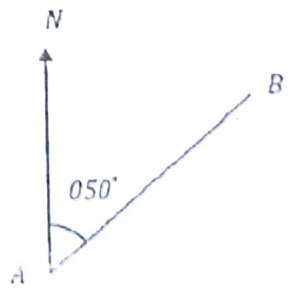
To find an angle when all the three sides are given:

- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
- $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

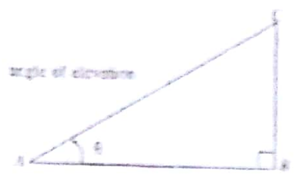
Bearing

The bearing of a point B from another point A is;

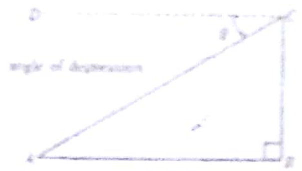
- an angle measured from the north at A.
- In a clockwise direction.
- Written as three-figure number (i.e. from 000 ° to 360°)
- The bearing of B from A is 050°.



Angles of elevation and depression

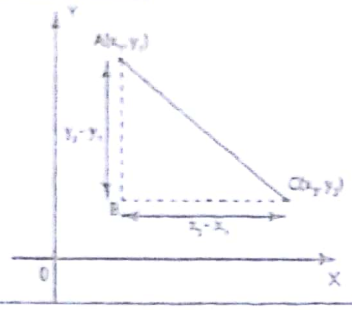


The angle of elevation (θ) of C from A is measured from the horizontal AB to the line AC



The angle of depression (θ) of A from C is measured from the horizontal CD to the line AC

Distance and Gradient



Distance Between Point A and C =

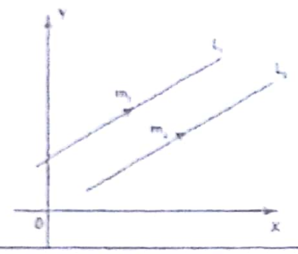
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Gradient of line AC, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Or

Gradient of a line, $m = -\left(\dfrac{y-\text{int ercept}}{x-\text{int ercept}}\right)$

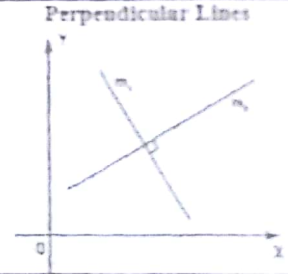
Parallel Lines



When 2 lines are parallel,

$$m_1 = m_2$$

Perpendicular Lines

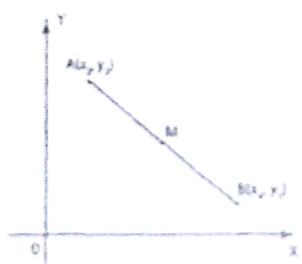


When 2 lines are perpendicular to each other,

$$m_1 \times m_2 = -1$$

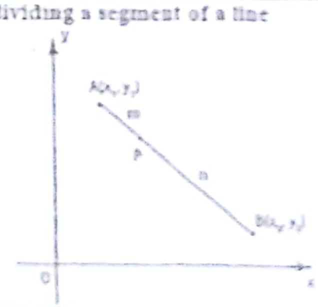
m_1 = gradient of line 1
m_2 = gradient of line 2

Midpoint



Midpoint, $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

A point dividing a segment of a line



A point dividing a segment of a line

$$P = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

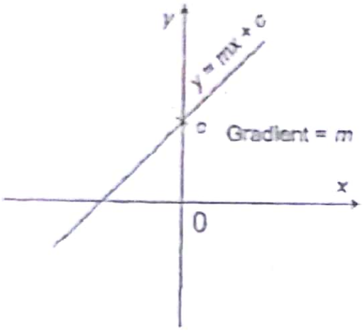
Cartesian co-ordinates

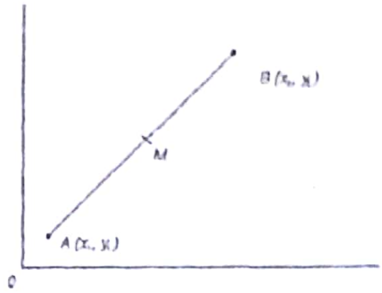
Gradient and equation of a straight line

The gradient of the straight line joining any two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is;

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The gradient/intercept form of the equation of a straight line is $y = mx + c$, where m = gradient and c = intercept on y – axis.



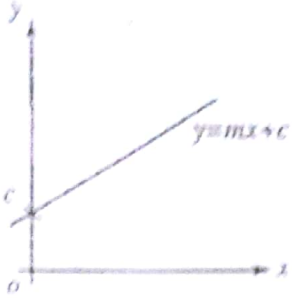


- The midpoint of the line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is; $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Parallel lines have the same gradient.
- In a graph, gradient = $\frac{\text{Vertical height}}{\text{Horizontal height}}$ or $\frac{y}{x}$
- Gradient point form of the equations

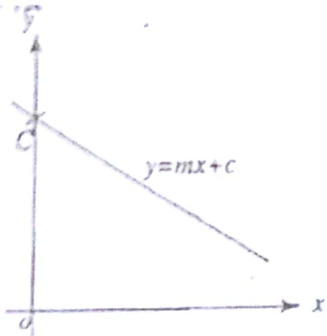
$$y - y_1 = m(x - x_1)$$

- Where m = gradient and $(x1, y1)$ is a point on the line

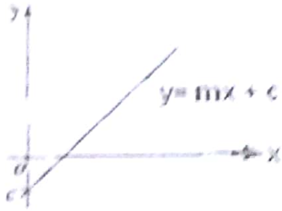
Gradients of some straight lines



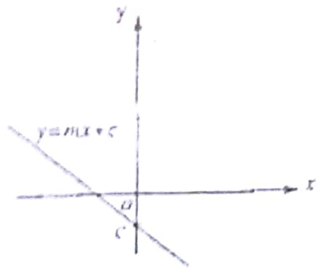
(a) $m > 0, c > 0$



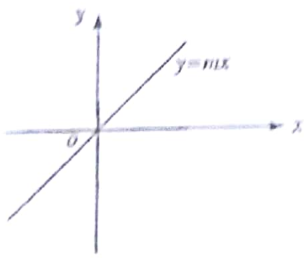
(b) $m < 0, c > 0$



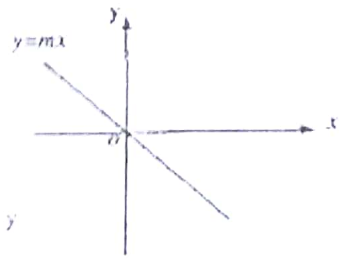
(c) $m > 0, c < 0$



(d) $m < 0, c < 0$

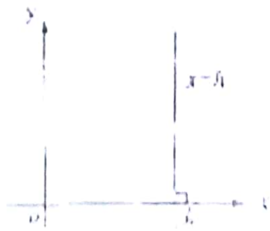


(e) $m > 0, c = 0$

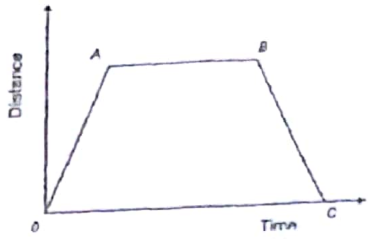


(f) $m < 0, c = 0$

(g) $m = 0$

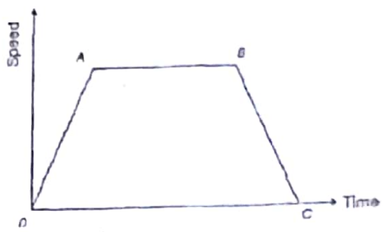


(h) m = undefined



Distance – Time Graphs:

From O to A : Uniform speed
From B to C : uniform speed
From A to B : Stationery (speed = 0)
The gradient of the graph of a distance-time graph gives the speed of the moving body.

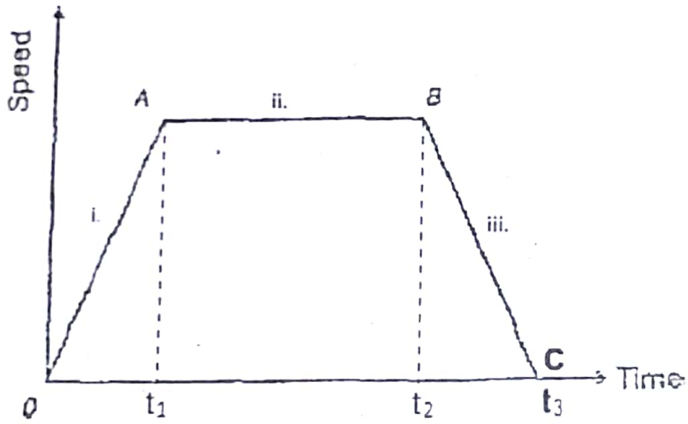


Speed – Time Graphs:

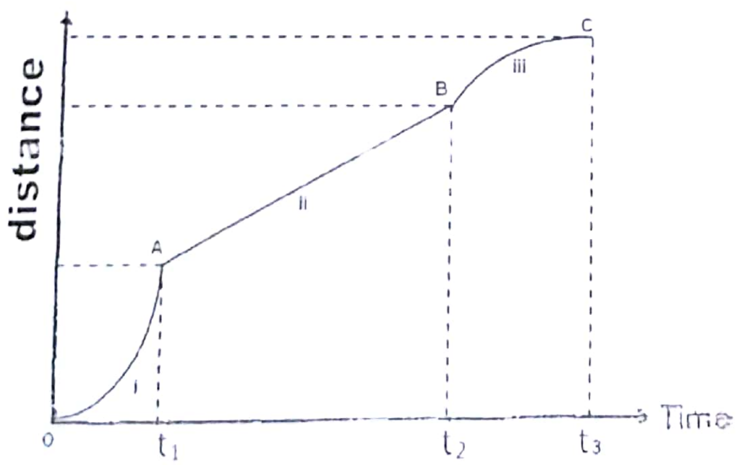
From O to A : Uniform speed (Uniform Acceleration)
From A to B : Constant speed (acceleration = 0)
From B to C : Uniform deceleration / retardation

The **area** under a speed –time graph represents the **distance travelled.**
The **gradient** of the graph is the **acceleration**. If the acceleration is negative, it is called deceleration or retardation. (The moving body is slowing down.)

Note: Conversion from speed time graph to distance time graph.



The Distance-Time Graph is given by:



Velocity:

Velocity is the rate of change of distance with respect to the time.

Acceleration:

Acceleration is the rate of change of velocity with respect to time.

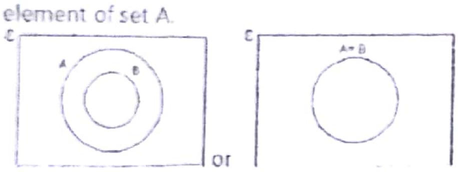
SETS:

Notations

- ξ = universal set
- ∪ (union) = all the elements
- ∩ (intersection) = common elements
- ∅ or { } = empty set
- ∈ = belongs to
- ∉ = does not belongs to
- ⊆ = Subset
- A' = compliment of A (i.e. the elements of ξ - the elements of A)
- n(A) = the number of elements in A.
- De Morgan's Laws: $(A \cup B)' = (A' \cap B')$
  $(A \cap B)' = (A' \cup B')$

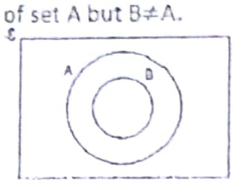
Subset ⊆

$B \subseteq A$ means every elements of set B is also an element of set A.



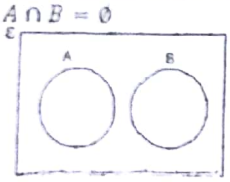
Proper subset ⊂

B ⊂ A means every element of B is an element of set A but B≠A.



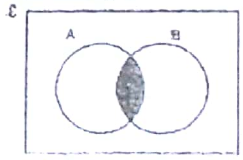
Disjoint sets

Disjoint set do not have any element in common. If A and B are disjoint sets, then $A \cap B = \emptyset$



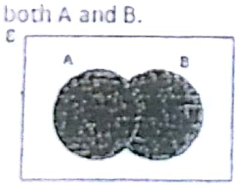
Intersection ∩

$A \cap B$ is the set of elements which are in A and also in B



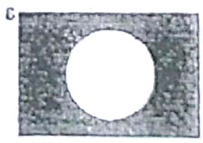
Union ∪

$A \cup B$ is the set of elements in either A , B or both A and B.



Complement

The complement of A, written as A' refers to the elements in ε but not in A.



- ## Number of Elements

The number of elements in set A is denoted by $n(A)$.

Notes

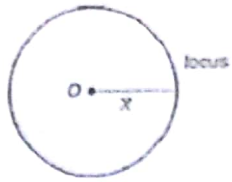
- $n(\emptyset) = 0$
- $n(A') = n(\varepsilon) - n(A)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Loci and construction

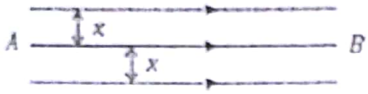
The locus of a point is a set of points satisfying a given set of conditions.

(a) Locus of points at a distance x from a given point, O.

Locus: *The circumference of a circle centre O, radius x.*



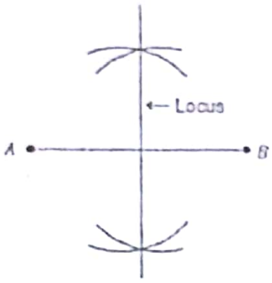
(b) Locus of a points at a distance x from a straight line AB



Locus: *A pair of parallel lines to the given line AB.*

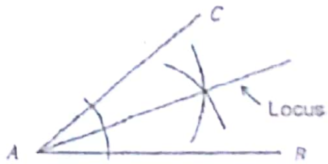
(c) Locus of points equidistance between 2 points.

Locus: *Perpendicular bisector of the two points.*



(d) Locus of points equidistant from two given lines AB and AC

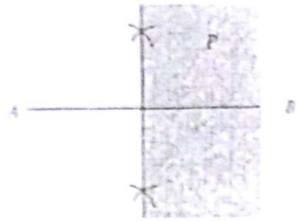
Locus: *Angle bisector of* $\angle BAC$



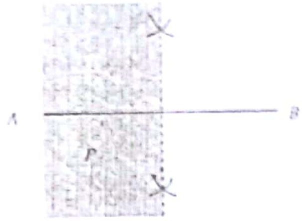
• Shading of Loci

(a) Shade the locus of P given that

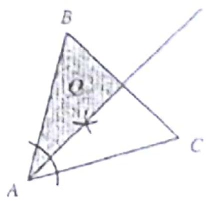
(i) $AP \geqslant PB$



(ii) $AP < PB$

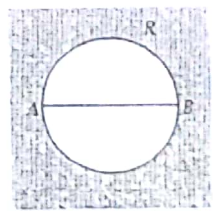


(b) Shade the locus of Q inside ΔABC given that $\angle BAQ < \angle CAQ$.

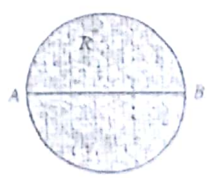


(c) Shade the locus of R given that

(i) $\angle ARB \leqslant 90°$

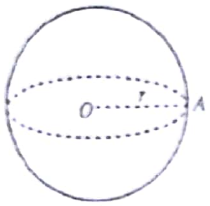


(ii) $\angle ARB \geqslant 90°$



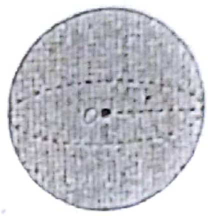
• Loci in Three Dimensions

(a)



The locus of points at a distance r from O is the surface area of a sphere, centre O, radius r.

(b)

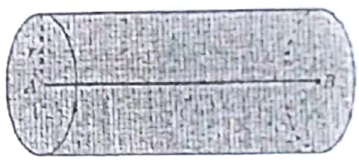


If $OA \leq r$, a solid sphere with centre O and radius r is obtained.

(c)

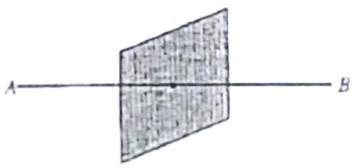
The locus of points at a distance r from a straight line, AB is the surface area of a cylinder, radius r and the line segment as its axis.

(d)



If the distance $\leq r$, then a solid cylinder, radius r and axis AB is obtained.

(e)

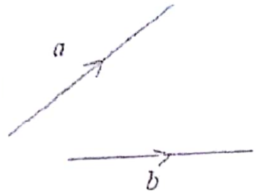


The locus of points which are equidistant from two points A and B is a plane which is perpendicular to AB and bisects AB.

Vectors:

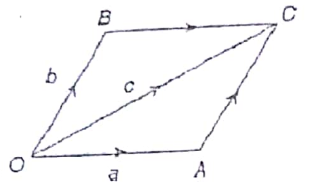
- A vector quantity has both magnitude and direction.

- Vectors a and b represented by the line segments can be added using the parallelogram rule or the nose- to- tail method.



- A scalar quantity has a magnitude but no direction. Ordinary numbers are scalars.
- The negative sign reverses the direction of the vector.
- The result of a – b is a +(-b)
  i.e. subtracting b is equivalent to adding the negative of b.

Addition and subtraction of vectors



$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ (Triangular law of addition)

$\overrightarrow{OB} + \overrightarrow{OA} = \overrightarrow{OC}$ (parallelogram law of addition)

Column Vectors:

The top number is the horizontal component and the bottom number is the vertical component $\begin{pmatrix} x \\ y \end{pmatrix}$

Parallel Vectors:

- Vectors are parallel if they have the same direction. Both components of one vector must be in the same ratio to the corresponding components of the parallel vector.
- In general the vector $k\begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to $\begin{pmatrix} a \\ b \end{pmatrix}$
- If two vectors $\underline{a}$ and $\underline{b}$ are parallel, then $\underline{a} = k\underline{b}$

Modulus of a Vector:

The modulus of a vector a, is written as $|a|$ and represents the length (or magnitude) of the vector.

In general, if $x = \begin{pmatrix} m \\ n \end{pmatrix}$, $|x| = \sqrt{(m^2 + n^2)}$

MATRICES:

Addition and Subtraction:

Matrices of the same order are added (or subtracted) by adding (or subtracting) the corresponding elements in each matrix.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a-p & b-q \\ c-r & d-s \end{pmatrix}$$

Multiplication by a Number:

Each element of a matrix is multiplied by the multiplying number.

$$k \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Multiplication by another Matrix:

Matrices may be multiplied only if they are compatible. The number of columns in the left-hand matrix must equal the number of rows in the right-hand matrix.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{pmatrix}$$

- In matrices A^2 means A × A. [you must multiply the matrices together]

The Inverse of a Matrix:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \frac{1}{(ad-bc)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

- $AA^{-1} = A^{-1}A = I$ where I is the identity matrix.
- The number (ad − bc) is called the determinant of the matrix and is written as $|A|$
- If $|A| = 0$, then the matrix has no inverse.
- Multiplying by the inverse of a matrix gives the same result as dividing by the matrix.

e.g. if $AB = C$

$A^{-1}AB = A^{-1}C$

$B = A^{-1}C$

- If $C = \begin{pmatrix} x \\ y \end{pmatrix}$ and $D = \begin{pmatrix} r \\ s \end{pmatrix}$ then $C + D = \begin{pmatrix} x+r \\ y+s \end{pmatrix}$

Transformations:

a) **Reflection:** [M]

When describing a reflection, the position of the *mirror line* is essential. [LOR-E]

b) **Rotation:** [R]

To describe a rotation, the *centre of rotation*, the *angle of rotation* and the *direction of rotation* are required.

A clockwise rotation is negative and an anticlockwise rotation is positive.

>> (angle) (Direction)rotation about (centre) [ADRC]

c) **Translation:** [T]

When describing a translation it is necessary to give the *translation vector* $\begin{pmatrix} x \\ y \end{pmatrix}$

- $+x$ represents movement to the right
- $-x$ represents movement to the left
- $+y$ represents movement to the top
- $-y$ represents movement to the bottom.

The Translation matrix equation is:

$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

where $\begin{pmatrix} a \\ b \end{pmatrix}$ = Traslation Matrix

$\begin{pmatrix} x \\ y \end{pmatrix}$ = Object

$\begin{pmatrix} x' \\ y' \end{pmatrix}$ = Image

>> Translation by the column vector ---- [T.C.V]

d) **Enlargement:**

To describe an enlargement, state;

i. The scale factor, K

ii. The centre of enlargement (the invariant point)

$$\text{Scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

>> Enlargement by the scale factor — centre ---- [E-SF-E]

- If $K > 0$, both the object and the image lie on the same side of the centre of enlargement.
- If $K < 0$, the object and the image lie on opposite side of the centre of enlargement.
- If the scale factor lies between 0 and 1, then the resulting image is smaller than the object. [although the image is smaller than the object, the transformation is still known as an enlargement]
- Area of image figure = k^2 × area of object figure

Repeated Transformations:
XT(P) means 'perform transformation T on P and then perform X on the image.'
XX(P) may be written X^2(P).

Inverse Transformations:
The inverse of a transformation is the transformation which takes the image back to the object.

If translation T has a vector $\begin{pmatrix} x \\ y \end{pmatrix}$, then the translation which ahs the opposite effect has vector $\begin{pmatrix} -x \\ -y \end{pmatrix}$.

This is written as T^{-1}.
If rotation R denotes 90° clockwise rotation about (0, 0), then R^{-1} denotes 90° anticlockwise rotation about (0, 0).
For all reflections, the inverse is the same reflection.

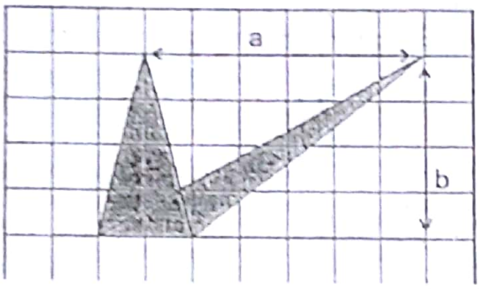
Base vectors

The base vectors are considered as $I = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $J = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The columns of a matrix give us the images of I and J after the transformation.

Shear: [S]

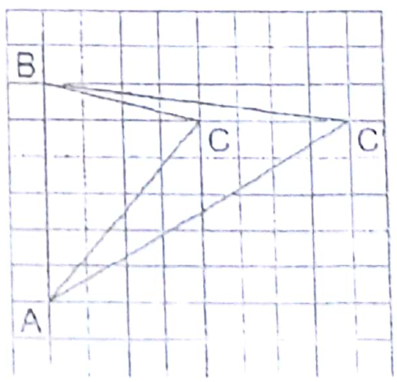
$$\text{Shear factor} = \frac{\textit{Distance a point moves due to the shear}}{\textit{Peapendicular distance of the point from the fixed line}} = \frac{a}{b}$$



[The shear factor will be the same calculated from any point on the object with the exception of those on the invariant line]

Area of image = Area of object

Stretch: [H] [S.F., I.L., DOH]



To describe a stretch, state,

i. the stretch factor, *p*
ii. the invariant line,
iii. the direction of the stretch
(always perpendicular to the invariant line)

$$\text{Scale factor} = \frac{\textit{Perpendicular distance of } C' \textit{ from } AB}{\textit{Peapendicular distance of } C \textit{ from } AB}$$

Where, P is the stretch factor

Area of image = p × Area of object

Transformation by Matrices

Reflection

Matrix	Transformation
$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	Reflection in the x-axis
$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	Reflection in the y-axis
$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$	Reflection in the line y = x
$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$	Reflection in the line y = - x

Rotation

Matrix	Angle	Direction	Centre
$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	90°	anticlockwise	(0, 0)
$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	90°	clockwise	(0, 0)
$\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$	180°	Clockwise/ anticlockwise	(0, 0)

Enlargement

$\begin{pmatrix}k & 0\\0 & k\end{pmatrix}$ where k = scale factor and centre of enlargement = (0, 0)

Stretch

Matrix	Stretch factor	Invariant line	Direction
$\begin{pmatrix}k & 0\\0 & 1\end{pmatrix}$	k	y-axis	Parallel to x - axis
$\begin{pmatrix}1 & 0\\0 & k\end{pmatrix}$	k	x - axis	Parallel to y - axis

Shear

Matrix	Shear factor	Invariant line	Direction
$\begin{pmatrix}1 & k\\0 & 1\end{pmatrix}$	k	x-axis	Parallel to x - axis
$\begin{pmatrix}1 & 0\\k & 1\end{pmatrix}$	k	y - axis	Parallel to y - axis

- **Note:** matrix for combining transformation is obtained by multiplying both matrices given.

STATISTICS

Bar Graph:

A bar chart makes numerical information easy to see by showing it in a pictorial form.

The width of the bar has no significance. The length of each bar represents the quantity.

Pie Diagram:

The information is displayed using sectors of a circle.

- **Angle of Sector** $= \dfrac{\text{Value of component}}{\text{Total value}} \times 360^\circ$
- **Value of Component** $= \dfrac{\text{Angle of Sector}}{360} \times \text{total value}$

Histograms:

A histogram displays the frequency of either continuous or grouped discrete data in the form of bars.

The bars are joined together.

The bars can be of varying width.

The frequency of the data is represented by the area of the bar and not the height.

[When class intervals are different it is the area of the bar which represents the frequency not the height]. Instead of frequency being plotted on the vertical axis, frequency density is plotted.

Frequency density = $\frac{frequency}{class\ width}$

Mean:

The mean of a series of numbers is obtained by adding the numbers and dividing the result by the number of numbers.

Mean = $\frac{\sum fx}{\sum f}$ where $\sum fx$ means 'the sum of the products'

i.e. $\sum$ (number × frequency)

and $\sum f$ means ' the sum of the frequencies'.

Median:

The median of a series of numbers is obtained by arranging the numbers in ascending order and then choosing the number in the 'middle'. If there are two 'middle' numbers the median is the average (mean) of these two numbers.

Mode:

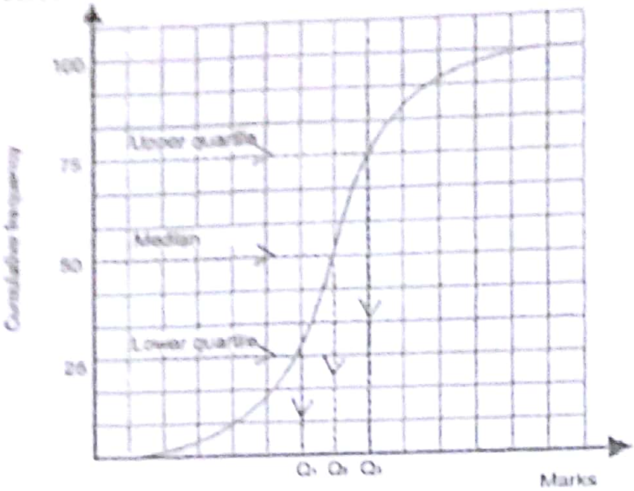
The mode of a series of numbers is simply the number which occurs most often.

Frequency tables:

A frequency table shows a number x such as a score or a mark, against the frequency f or number of times that x occurs.

Cumulative frequency:
Cumulative frequency is the total frequency up to a given point.

Cumulative frequency Curve:



A cumulative frequency curve shows the median at the 50th percentile of the cumulative frequency. The value at the 25th percentile is known as the lower quartile and that at the 75th percentile as the upper quartile.
A measure of the spread or dispersion of the data is given by the inter-quartile range where
Inter-quartile range = upper quartile – lower quartile.

Probability:

- Probability is the study of chance, or the likelihood of an event happening.
- Probability of an event = $\frac{\text{number of favourable outcomes}}{\text{Total number of equally likely outcome}}$
- If the probability = 0 it implies the event is impossible
- If the probability = 1 it implies the event is certain to happen.
- All probabilities lie between 0 and 1.
- Probabilities are written using fractions or decimals.

Exclusive events:
Two events are exclusive if they cannot occur at the same time.
The OR Rule:
For exclusive events A and B
$p(A \text{ or } B) = p(A) + p(B)$

Independent events:
Two events are independent if the occurrence of one even is unaffected by the occurrence of the other.
The AND Rule:
$p(A \text{ and } B) = p(A) \times p(B)$
where $p(A)$ = probability of A occurring
$p(B)$ = probability of B occurring

Tree diagrams:
A tree diagram is a diagram used to represent probabilities when two or more events are combined

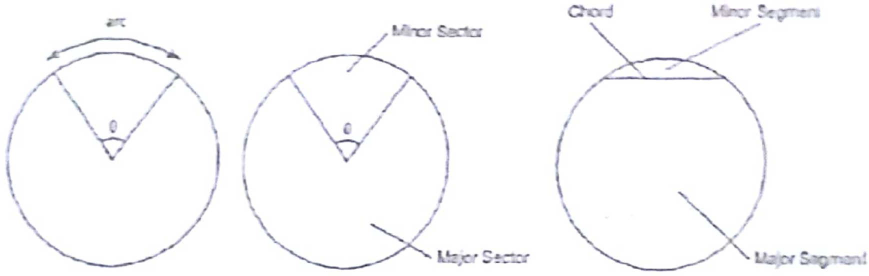
Symmetry:

- A line of symmetry divides a two-dimensional shape into two congruent (identical) shapes.
- A plane of symmetry divides a three-dimensional shape into two congruent solid shapes.
- A two-dimensional shape has rotational symmetry if, when rotated about a central point, it fits its outline. The number of times it fits its outline during a complete revolution is called the order of rotational symmetry.

Shape	Number of Lines of Symmetry	Order of Rotational Symmetry
Square	4	4
Rectangle	2	2
Parallelogram	0	2
Rhombus	2	2
Trapezium	0	1
Kite	1	1
Equilateral Triangle	3	3
Regular Hexagon	6	6

- **Note:**
  Number of Planes of Symmetry = n + 1, where n is the number sides of the base of the prism/three dimensional figure.

Terminology

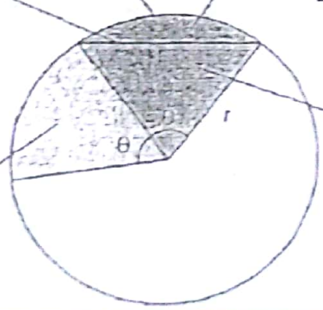


Length and Area

Length of Arc: $s = r\theta$

Area of Segment: $A = \frac{1}{2}r^2(\theta - \sin\theta)$

Length of Chord: $l = 2r\sin\frac{\theta}{2}$



Area of Triangle: $A = \frac{1}{2}r^2\sin\theta$

Area of Sector: $A = \frac{1}{2}r^2\theta$

r = radius
A = area
s = arc length
θ = angle
l = length of chord

Arc Length:	Length of chord:	Area of Sector:	Area of Triangle:	Area of Segment:
$s = r\theta$	$l = 2r\sin\frac{\theta}{2}$	$A = \frac{1}{2}r^2\theta$	$A = \frac{1}{2}r^2\sin\theta$	$A = \frac{1}{2}r^2(\theta - \sin\theta)$

Formula for Compound interest

Total Amount with interest, $A = P\left(1 + \frac{P}{100}\right)^{N}$

Where A is

Amount with interest, if we want to calculate interest, then

I = A — P

Where

I = Interest

P = Principal amount

R = Rate of interest

N = number of years

Formulae to find nth term for a number sequence

(i) Tn = a+ (n—1) d, where

a = first term of the sequence

d= the common difference

n = number of terms

The above formula is used, when there is a common difference between the terms of a sequence.

(ii) Tn= a r^{n-1}, where

a = first term of the sequence

r= the common ratio

n = number of terms

the above formula is used, when there is a common ratio between the terms of a sequence.

(iii) Finding the nth term when there is no common difference or common ratio between the terms

$$T_n = a + (n-1)d + \frac{(n-1)(n-2)}{2} \times C$$

Where

a = first term of the sequence

d= the difference between first two terms

n = number of terms

c = the continue difference

(iv) Trial and error method

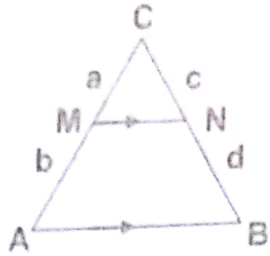
Note:

(i) Ratio of mass = ratio of volumes

$$\frac{\text{mass of larger sphere}}{\text{mass of a smaller sphere}} = \frac{\text{vol of larger sphere}}{\text{vol of a smaller pheres}} = \left(\frac{d1}{d2}\right)^3$$

(ii) A straight line drawn parallel to one side of a triangle divides the other two sides proportionally, conversely, if a line divides two sides of a triangle proportionally, then it is parallel to the 3rd side i.e.

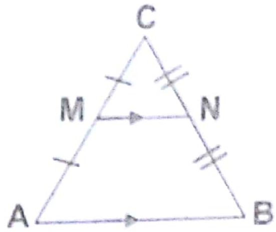
if MN || AB $\Leftrightarrow \frac{a}{b} = \frac{c}{d}$



(iii) The line joining mid-points of any two sides of a triangle, always parallel to the third side and half in length to the 3rd side.

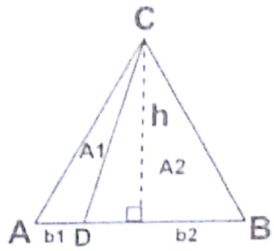
if M and N are mid-points of AC and BC respectively,

Then, MN || AB, MN = $\frac{1}{2}$ AB



(iv) For Δs, of same height

$$\frac{A1}{A2} = \frac{b1}{b2}$$



Note! The ⊥ distance d of a line ax + by + c = 0 from a point c (x_1, y_1)

Is, ⊥ distance $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

The Ratio Formula

Let A and B be the two points whose position vectors are a and b respectively. If a point P divides AB in the ratio P:q, then the position vector of P is given by $\mathbf{P} = \frac{q\underline{a} - p\,\underline{b}}{p - q}$

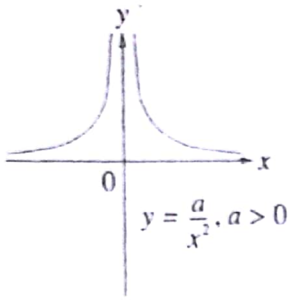
Note! If P is the mid-point of AB, then $p:q = 1:1$

Then the position vector of p is $\underline{r} = \frac{a + b}{2}$

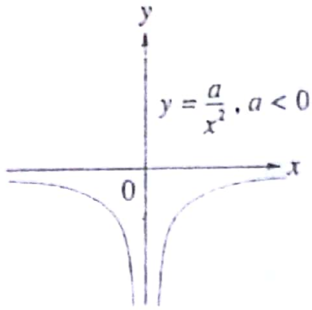
Sketches of Some Important Graphs

The following are sketches of the functions with the form $y = ax^n$, where $n = -2, -1, 1, 2$ and 3.

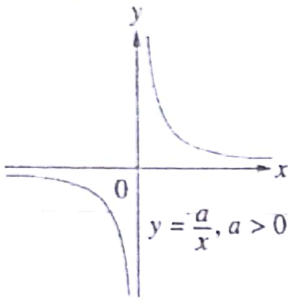
(a) $a > 0, n = -2$



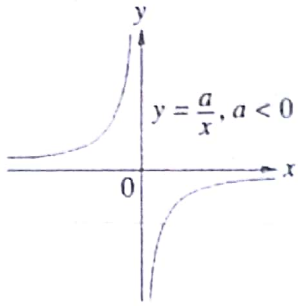
(b) $a < 0, n = -2$



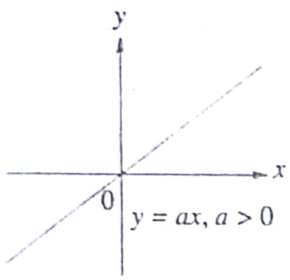
(c) $a > 0, n = -1$



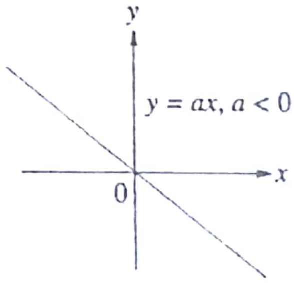
(d) $a < 0, n = -1$



(e) $a > 0, n = 1$

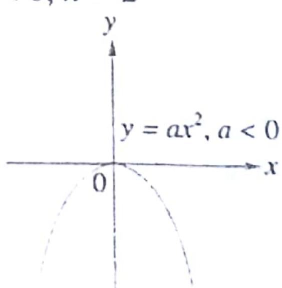


(f) $a < 0, n = 1$

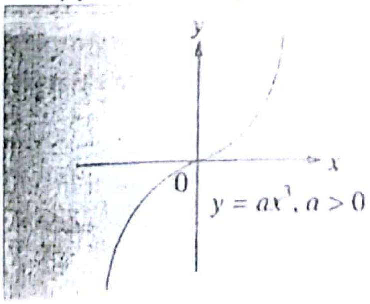


(g) $a > 0, n = 2$

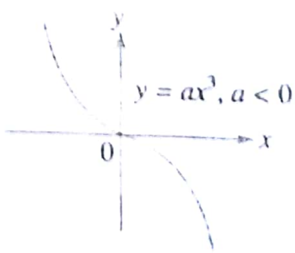
(h) $a < 0, n = 2$



(i) $a > 0, n = 3$



(j) $a < 0, n = 3$



FORMULAE AND IMPORTANT NOTES

1. The general form of a quadratic function is $y = ax^2 + bx + c$. One or both b and c may be zero.
2. Draw a horizontal line segment. This is the x-axis. Label it.
3. Put $y = 0$ and solve for x. If there is no real roots go to No. 8. Mark and label the roots (or root) on the x-axis. They are the x-intercepts.
4. Draw a line segment perpendicular to the x-axis at the origin to form a cross. This is the y-axis. Label it.
5. Put $x = 0$ and solve for y. Mark the value on the y-axis. It is the y-intercept.
6. If a is positive the graph is in the shape of an upright bowl. If a is negative the graph is in the shape of an inverted bowl. These curves are called parabolas. A parabolic curve is symmetrical about the vertical line through the vertex of the curve. The vertex is called the turning point. It is a minimum point if $a>0$ and a maximum point if $a<0$.
7. Join the points of No. 3 and No. 5 with a smooth parabola. The axis of symmetry must pass through the x-intercept if there is only one, or halfway between them if there are two. The axis of symmetry needs not be drawn unless asked but the graph must be symmetrical about it. Write the quadratic function above the graph.
8. By 'completing the square' any quadratic function can be expressed as $a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$. The point $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$ is a maximum point if $a<0$ and a minimum point if $a>0$. The y-intercept is c. If $b^2-4ac<0$ the quadratic equation $ax^2+bx+c=0$ has no real root. When this happens the minimum point and y-intercept are above the x-axis and the maximum point and y-intercept are below the x-axis.
9. If b^2-4ac is zero there is only one x-intercept $-\frac{b}{2a}$. If it is positive there are two x-intercepts.
10. To sketch the graph of $|ax^2+bx+c|$: sketch the graph of ax^2+bx+c. Draw the reflection image of the portion below the x-axis, if any. Erase the portion of the graph below the x-axis.

Geometry of Matrices

Transformations	Matrices	Properties
Reflection in x-axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Figures transformed into congruent figures (distances, angle, area, shape and size preserved)
Reflection in y-axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	
Reflection in y = x	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	
Reflection in y = – x	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	
Rotation through 90°, counterclockwise	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	
Rotation through 270°, counterclockwise or 90° clockwise	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	
Rotation through 180°	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	
Translation	-----	
Enlargement	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	Figure transformed into similar figure (angle and shape preserved; area increased by k^2 times)
Shear parallel to x-axis	$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$	x-axis invariant; Area preserved
Shear parallel to y-axis	$\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$	y-axis invariant; Area preserved
Stretching parallel to x-axis	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$	y-axis invariant
Stretching parallel to y-axis	$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$	x-axis invariant

Stretch:

1. A stretch has, in general, the transformation matrices as shown in the table below

Stretch Matrix	Direction of Stretch	Stretch Factor	Invariant Line
$\begin{pmatrix} k_1 & 0 \\ 0 & 1 \end{pmatrix}$	parallel to x-axis	k_1	y-axis
$\begin{pmatrix} 1 & 0 \\ 0 & k_2 \end{pmatrix}$	parallel to y-axis	k_2	x-axis
$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$	parallel to x-axis	k_1	y-axis
	parallel to y-axis	k_2	x-axis

2 Under a stretch with the line l as the invariant line and stretch factor k, if A' is the image of A under the stretch, then

$$k = \frac{\text{distance of } A' \text{ from } l}{\text{distance of A from } l}$$

3. For a stretch parallel to x- and y-axes with scale factor k_1 and k_2 respectively, image area = $k_1k_2 \times$ (original area).

Shear:

1. A shear preserves area and has, in general, the transformation matrices as shown in the table below.

Shear Matrix	Direction of Shear	Shear Factor	Invariant Line
$\begin{pmatrix} 1 & k_1 \\ 0 & 1 \end{pmatrix}$	along x-axis	k_1	x-axis
$\begin{pmatrix} 1 & 0 \\ k_2 & 1 \end{pmatrix}$	along y-axis	k_2	y-axis

2. Under a shear with the line l as the invariant line and shear factor k, if A' is the image of A under the shear, then

$$k = \frac{\text{distance of A from } A'}{\text{distance of A from } l}$$

3. A shear is a non-isometric transformation that preserves the area of the figure.

Remember these:-

Reflection:

1. Reflection carries segments into equal segments.
2. Reflection carries angles into equal angles.
3. Reflection carries whole figures into congruent whole figures.
4. It has invariant points if some points on the original figure fall on the mirror line.

Rotation:

1. A rotation carries segments into equal segments.
2. A rotation carries angles into equal angles.
3. A rotation carries figures into congruent figures.
4. The mediators (perpendicular bisectors) of the line segments joining the points and their images meet at the centre of rotation.
5. It has only one invariant point, provided the centre of rotation lies on the original figure.

Translation:

1. Translation carries segments into equal and parallel segments.
2. Translation carries angles into equal angles.
3. Translation carries whole figures into congruent whole figures.
4. It has no invariant points.

Enlargement:

1. Enlargement is clearly defined by its centre and scale factor.
2. Under enlargement, each length of the image of the figure is $|k|$ times the original length where $|k|$ is the numerical value of k, the enlargement factor (scale factor).
3. If $|k| > 1$, the image is magnified in size.
4. If $|k| < 1$, the image is reduced in size.
5. If the enlargement factor is positive, the figure and its image lie on the same side of the centre of enlargement.
6. If the enlargement factor is negative, the figure and its image lie on the opposite sides of the centre of enlargement, and the image is inverted.
7. Enlargement carries angles into equal angles.
8. Enlargement carries whole figures into similar figures.
9. The centre of enlargement is the only possible invariant point.
10. Under an enlargement with scale factor k and centre at the origin, the matrix representations is $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$.
11. Image area = $k^2 \times$ (original area).